CLOUD WATER CONTENT AND CLOUD PARTICLE CHARACTERISTICS REVEALED BY DUAL WAVELENGTH CLOUD RADAR OBSERVATIONS

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1 INTRODUCTION

Observations of clouds made with a single frequency radar are difficult to interpret in terms of water content and particle size, shapes and densities, but most of these ambiguities can be removed if two frequencies are used.

In stratocumulus and liquid water clouds, profiles of liquid water content can be derived from the increased attenuation by liquid water at 94GHz. In ice clouds the particle size can be inferred from the reduced reflectivity, $Z$, due to Mie scattering at 94GHz when compared to the 35GHz reflectivity. The dual-wavelength difference of the Doppler velocities can be combined with the dual wavelength reflectivity ratio to derive the apparent density variation of ice particles with their size. Finally, when size and density informations are combined with the 35GHz reflectivity, more accurate ice water content can be derived, then those inferred from empirical relationships expressing IWC as a function of $Z$.

In this paper we present results obtained using measurements from the Cloud Profiling Radar System (CPRS) a dual-wavelength radar at 33- and 95-GHz at the central Atmospheric Radiation Measurement (ARM) site in Oklahoma.

2 STRATOCUMULUS LIQUID WATER CONTENT

The main difficulty in measuring LWC in liquid cloud is that they often contain small concentrations of ‘drizzle’ drops (those larger than around 50 µm) which can dominate the radar reflectivity but contribute negligibly to LWC (Fox and Illingworth 1997). Dual-wavelength measurements take advantage of the fact that small droplets dominate the attenuation and are small enough to attenuate in the Rayleigh regime.

2.1 Theory

In conventional logarithmic units the reflectivity measured at frequency $f$ and height $z$ may be given by

$$Z_f = Z_0 + 10 \log_{10} \left( \frac{[K_f]^2}{0.93} \right) - 2 \int_0^z (\alpha_f + \kappa_f \text{LWC}) \, dz,$$

where $Z_0$ is the unattenuated reflectivity factor at centimetre wavelengths in conventional dBZ units, $\alpha_f$ is the one-way specific attenuation coefficient due to atmospheric gases (predominantly molecular oxygen and water vapor) in dB km$^{-1}$, and $\kappa_f$ is the one-way specific attenuation coefficient of liquid water and has the units dB km$^{-1}$(g m$^{-3}$)$^{-1}$. The second term on the right hand side accounts for the fact that at millimeter wavelengths the dielectric parameter of water, $[K]^2$, is less than its centimeter-wavelength value of 0.93, and is a function of temperature. $K$ is related to the complex dielectric constant, $\varepsilon$, by $K = (\varepsilon - 1)/(\varepsilon + 2)$. We have assumed that extinction by absorption dominates extinction by scattering. We define the dual-wavelength ratio in logarithmic units as $\text{DWR} [\text{dB}] = Z_{94} [\text{dBZ}] - Z_{35} [\text{dBZ}]$. From (1), the mean LWC in a layer between heights $z_1$ and $z_2$ can then be determined from the DWR measured at the top and bottom of the layer (indicated by the subscripts 2 and 1 respectively):

$$\text{LWC} = \frac{1}{\kappa_{94} - \kappa_{35}} \left( \frac{\text{DWR}_1 - \text{DWR}_2 - \beta}{2(z_2 - z_1)} \right) - \alpha_{94} + \alpha_{35},$$

where

$$\beta = 10 \log_{10} \left( \frac{[K_{35}(T_1)]^2}{[K_{35}(T_2)]^2} \frac{[K_{94}(T_1)]^2}{[K_{94}(T_2)]^2} \right) \, \text{dB.}$$

It has been assumed that the attenuation coefficients $\alpha$ and $\kappa$ are constant over the depth of the layer. The $\beta$ parameter accounts for the difference in temperature $T$ at the two heights.

The attenuation coefficients and the $\beta$ parameter are functions of temperature, and the attenuation of atmospheric gases is also dependent on pressure and humidity, so an independent measure of the vertical profiles of these variables is required in the retrieval. These can be obtained from radiosonde or from the output of a forecast model, although in practice it is always assumed that cloudy air is saturated with respect to liquid water. We calculate $\varepsilon$ (and hence $\beta$ and $\kappa$) from temperature using the formulation of Liebe et al. (1989), and the line-by-line model of Liebe (1985) to compute $\alpha$ as a function of temperature, pressure and humidity.

2.2 Uncertainties

The accuracy which LWC can be determined depends on the precision of the reflectivity measurements, the accuracy of the temperature profile and the overlap precision of the sample volumes of the radars. It can be shown (Hogan 1998) that the errors in reflectivity are related to
the temporal and vertical averaging and to signal-to-noise ratio. Errors due to Mie scattering effects are smaller than 0.05 g m\(^{-3}\) if drizzle drops are smaller than 600 \(\mu\)m. Thus, lidar cloud base is used to screen out data contaminated by large drizzle drops falling below cloud base. Uncertainties due to errors in temperature profile are found to be very small. We assume that errors due to cloud inhomogeneity are negligible, since the CPRS is built on a single feed and single lens antenna that reduce pointing errors and since at least a minute averaging is applied to the data.

2.3 Results

In Figure 1, observations are presented from the CPRS on the 23 September 1997. The reflectivity field in Fig.1a shows a complex structure with more than one layer present. Drizzle drops larger than 600 \(\mu\)m will Mie scatter and start to make a contribution to the DWR, but their terminal velocity will be 2.5 m s\(^{-1}\), rather larger than the 2 m s\(^{-1}\) measured in Fig. 1b at 21:07. So, we believe that this effect is negligible and after calculating DWR and deriving LWC (Fig.1c and 1d), this assumption is confirmed by the excellent agreement of values of the vertically-integrated LWC presented in Fig.1e with the LWP estimated from microwave radiometer. Note that the values of LWC bear little relationship to \(Z\), confirming the dangers of applying a simple \(Z\)-LWC law.

3 ICE PARTICLE DENSITY AND SIZE

Brown et al. (1995) have used aircraft measurements to get values of the mass and the maximum dimension of the ice particles and they found that \(\rho(D) = 0.07D^{-1.1}\) where \(D\) is the diameter of the equivalent spherical particle in millimeter. Using the same dataset, Francis et al. (1998) derived an alternative formula by expressing the diameter of a circle of area equal to the observed cross-section area and suggested that \(\rho(D) = 0.175D^{-0.66}\). But, the choice between these two density functions remains unclear and raise uncertainties in the estimation of size and IWC (Liu and Illingworth 2000).

3.1 Background

In ice clouds, the particle size-distribution is usually well represented by a simple exponential distribution \(N(D) = N_0 \exp(-\Lambda D)\) where \(\Lambda\) is related to the median volume diameter by \(D_v = 3.67/\Lambda\). The unattenuated reflectivity factor \(Z_0\), defined in Sec. 2, in the Mie scattering theory depends on the frequency of the radar and is given by:

\[
Z_0 = \int D^6 f(D, \lambda) N(D) dD, \tag{4}
\]

where \(f(D, \lambda)\) is the ratio of the Mie scattering to an assumed Rayleigh scattering for a given value of \(\lambda\). If large ice particles are present Mie scattering will occur for mm wave radar and \(f(D, \lambda)\) fall below unity. As a result, in the liquid absence of attenuation, DWR defined in section 2 is a function of \(D_v\) and can be used (Sekelsky et al. 1999, Hogan et al. 2000) to infer particles size. However Fig. 2a shows that the results also depend significantly on the density relations used.

![Figure 1: Dual-wavelength measurements by the CPRS on 23 September 1997: (a) radar reflectivity factor at 95 GHz, ceilometer cloud base (dashed line) and LWC=0.6 g m\(^{-3}\) (solid contour) (b) Doppler velocity and ceilometer cloud base, (c) dual wavelength ratio above the ceilometer cloud base, (d) liquid water content (e) comparison of the liquid water paths deduced from the dual-wavelength technique and microwave radiometers. The data were averaged to 1 minute and 60 m (2 gates) in vertical before liquid water content was derived.](image1)

![Figure 2: Theoretical results for Brown and Francis (1995) (plain lines) and Francis et al. (1998) (dotted lines) density relations, (a) DWR against \(D_v\), (b) \(\Delta V_D\) against \(D_v\), (c) \(\Delta V_D\) against DWR. The Doppler velocity measured by a vertically pointing radar is \(V_D = V_a + V_z\) where \(V_a\) is the vertical air velocity](image2)
and \( V_z \) the reflectivity-weighted particle terminal velocity given by:

\[
V_z = \frac{\int v_t(D)D^b f(D, \lambda)N(D) dD}{\int D^b f(D, \lambda)N(D) dD}, \tag{5}
\]

where \( v_t \) is the terminal velocity of an individual particle. \( D \) is usually given as the maximum particle dimension. Since in the following developments we assume that particles are spherical then \( D \) is the diameter. A basic equation to estimate the terminal velocity of ice particles is (Mitchell 1996):

\[
v_t = a \left( \frac{gA}{\rho_a} \right)^{\frac{b}{2}} v^{1-\frac{b}{2}} D^{b-1} \left( \frac{m}{A} \right)^{\frac{b}{a}} \tag{6}
\]

where \( g \) is the gravitational acceleration; \( \rho_a \) is the air density; \( v \) is the kinematic viscosity; \( m \) is the particle mass; \( A \) is the particle cross-section area; and \( a \) and \( b \) coefficients are derived from the drag law. Considering the spherical assumption and a density relationship lead to two distinct curves.

### 3.2 Observations

In March 2000, a long serie of cirrus cloud observations was performed by the CPRS. Figure 3 displays in the two first panels the reflectivity factor distribution and mean Doppler velocity distribution observed by the 33 GHz channel on the 12 March 2000 in 4 km deep ice cloud. The reflectivity field structure is dominated by convective motions at the top and the bottom of the cloud. It is difficult to find any kind of correlation between these two fields. In contrast the differential parameters DWR and \( \Delta V_z \) shown in the two panels underneath are very well correlated, the similar increases in both DWR and \( \Delta V_z \) are corresponding to growing values of \( D_0 \) related to aggregation processes. \( \Delta V_z \) as a function of DWR from the co-located measurements is plotted Fig.4. On top of the scatter plot, are the theoretical curves from Fig. 2c, for the two density functions. It can be seen without ambiguity that the scatter plot of data matches the theoretical calculations only if the Brown and Francis density relationship is used.

![Figure 3: Observations of cirrus by the CPRS on the 12 March 2000: (a) Reflectivity factor at 33 GHz, (b) Doppler velocity measured by the CPRS at 33 GHz (c) Dual-wavelength ratio, (d) Dual-wavelength Doppler velocity difference.](image1)

![Figure 4: \( \Delta V_z \) as function of DWR for both theoretical calculations and measurements](image2)

### 3.3 Ice water content calculations

Since the density relationship ambiguity has been resolved, a better estimation of \( D_0 \) is obtained from dual-wavelength measurements, and using measurements of the reflectivity at 35 GHz, a more accurate calculation of IWC can be produced (Hogan et al. 2000). Figure 5a and 5b shows values \( D_0 \) and IWC retrieved from the observations in Fig. 3. Figure 5c shows the IWC derived with IWC derived from IWC=0.137Z^0.64 empirical relation based on Brown and Francis density relationship assumption (Liu and Illingworth 2000). In Fig. 6 are plotted profiles of IWC at 23:42 obtained using both dual-wavelength and empirical methods and for the two density relationships.
2.5
3
3.5
4
4.5
5
5.5
6

Range [km]

(a) Median equivolumetric diameter

(b) Ice water content from dual-wavelength ratio

(c) Ice water content from Z-IWC relationship

Figure 5: Dual-wavelength retrieved parameters between 23:33 and 23:44: (a) Median equivolumetric diameter $D_0$, (b) Ice water content using DWR technique and Brown and Francis (1995) density relation, (c) Ice water content using $IWC=0.137Z^{0.643}$ empirical relation.

It can be seen that empirical and dual-wavelength derived IWC profiles based on Brown and Francis (1995) assumption are matching whereas those based on Francis et al. (1998) assumption disagree and also underestimate IWC by 30% to 60%.

4 CONCLUSION

In this paper, we have shown that from dual-wavelength radars measurements in liquid clouds, detailed vertically resolved measurements of LWC can be derived. Because drizzle is very often present in such clouds, the differential technique is successful when other technique fail. Mie scattering bias when drizzle is present does not appear to be a problem. Dual-wavelength measurements of Doppler velocities are also used to derive the density of ice particles as a function of size. A theoretical relation between the dual-wavelength reflectivity ratio and the dual-wavelength Doppler velocity difference is presented for two density functions. Comparison between observation and theoretical differential Doppler calculations agreed only when Brown and Francis density function is used. In addition, the IWC calculated from dual-wavelength was found to be very good agreement with the empirical Z-IWC relation only when Brown and Francis density function is used.

REFERENCES


Figure 6: Profiles of ice water content at 23:42 derived from DWR technique (in black) and Z-IWC empirical technique (in gray) for Brown and Francis (1995) (plain lines) and Francis et al. (1998) (dotted lines) density relations.

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